THE SPECTRAL STRUCTURE OF THE PROBLEM OF MAGNETIC ELASTICITY OF THIN PLATES

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Based on asymptotic analysis, approximate methods of solution of the problem of free oscillators of magnetically elastic plates of a finite size [1,2] are studied. The asymptotic properties of the spectrum of eigenvalues of the problem are formulated as a function of magnetic field intensity.

1. We proceed from the following equations, assuming that oscillations occur as (exp (iωt) (t is time):

\begin{align}
\frac{1}{i} \left[ \frac{\partial^2 u_i}{\partial x_i^2} + i \beta B_{\alpha \beta} \left( \frac{\partial^2 u_i}{\partial x_i} \right) \right] + \frac{1}{i} \left[ \frac{\partial^2 \Phi}{\partial x_i^2} \right] &= 0, \quad i=1,2,3 \\
\frac{1}{i} \left[ \frac{\partial^2 u_i}{\partial x_i^2} + \frac{1}{2i} \left( \frac{\partial^2 \Phi}{\partial x_i} \right) \right] &= 0, \quad i=1,2,3 \\
\frac{1}{i} \left[ \frac{\partial^2 u_3}{\partial x_3^2} + \beta B_{\alpha \beta} \left( \frac{\partial^2 u_3}{\partial x_3} \right) \right] &= 0, \quad \beta = 2 \alpha_0 \mu_0 E_0^2 / E, \quad \alpha_0 = \left[ 3(1-\nu)^2 \right]^{1/2}, \quad \beta = 2 \mu_0 \mu_4, \quad \frac{\lambda}{\mu_4} \left( 2h \mu_4 \right)^{1/2} \approx 1
\end{align}

Here k, j = 1, 2 are indexes (summation is performed over recurring indexes; the first equality is equivalent to two equations obtained at k = 1 and k = 2); x₁, x₂, x₃ are parameters of the triorthogonal coordinate system defined in the space V surrounding the plate, where the median surface S of the plate is coincident with the coordinate surface x₃ = 0, B₁, B₂, B₃ are values of the vector of magnetic induction on S (B₀ is assumed to be known from the solution of the magnetostatic problem and its components to differ from identical zero); uᵢ is the vector of displacements of the median surface of the plate; ϕ is a potential function (the disturbed component of the magnetic induction b = grad ϕ); the subscript s indicates that the value of the respective variable is taken on the surface S (at x₃ = 0); (..),* = (..)s; (bᵢ), is the normal component of b in the plate (which, according to [1,2] is assumed to be constant with respect to thickness); Δ and Δₓ are the three-dimensional (in V) and two-dimensional (on S) Laplacians, respectively; Lᵏᵢ are familiar operators of the theory of elastic plates; h is the half-thickness of the plate; μ₀ is the magnetic constant; α, ρ, ε, ν are electric conductivity, density, Young's modulus, and Poisson's ratio of the plate material; λ is the frequency parameter; and Jᵢ = Jᵢₙ are arbitrary factors introduced for convenience (provisionally, they are set equal to 1).

Equations (1.1)-(1.5) are derived from the equations of [1,2], excluding the tangential components of the electric field induced in the plate and omitting the terms that are beyond the accuracy limits of the Kirchhoff-Love hypotheses. The magnetic permeability constant is set equal to 1. In the analysis of an induced magnetic field in the medium no difference is made between the face surfaces and the median surface of the plate.

The thickness-averaged vectors of the electric current I and the electric field Φ induced in the plate can be obtained from the pertinent formulas of [1,2], and written in terms and with accuracy of (1.1)-(1.5):

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I = \frac{1}{2\mu_0} \left[ -\frac{\partial}{\partial x_1} (F-2kf) \right] + \frac{\partial}{\partial x_1} (F-2kf) \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] \right] 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sides of the equalities are of the order of $O(n^0)$. The number $g$ then defines the asymptotic behavior of the intensity of a magnetic field with respect to $(2E\mu)^0$. The number $p$ coincides in its sense with the variability indicator $[3]$, and $r$ determines the asymptotics of the frequency parameter $\lambda$. The numbers $p$ and $r$ ought to satisfy the inequalities:

$$0 < p < 1, \quad \max (-1, a-2) < r = \infty$$

which define the applicability of two-dimensional shell theory $[4]$ and the hypotheses of magnetic elasticity $[1]$. The last inequality is equivalent to the condition $(2h)^{-p} / a < 1$ of the absence of a skin effect in the plate. In addition, each of the six approximate solutions listed under (2.4) holds, accurate to terms that are asymptotically small compared with those retained in (2.4), only within the variation region of the parameters $r, p, g$ defined by the expressions

1. $r = -p, g < 1/(p-a) \quad (a<1), g < 1/(p-1) \quad (a\geq 1)$
2. $r = 1-2p, g < 1/(1+a) \quad (p<2-a), g < 1/(p-1) \quad (p\geq 2-a)$
3. $r = -1-p, g < 1/(3-3p) \quad (1-2a+p)$
4. $r = -1/(1+p) - g, r < \min (a-1-p, 1-2p), r > p$
5. $r = -2g, a-1-p = \leq 1-2p, r > p$
6. $r = 2+p+2g-4p, r > \max (1-2p, a-1-p)$

In expressions (2.4), we assume that $j' = 0$ at $r > -p$ and $j' = 1$ at $r < -p$.

In the three-dimensional space of parameters $(p, r, g)$, expressions (2.7) and (2.8) determine the plates defined by the first equalities of (2.8) and limited by inequalities (2.7) and the corresponding inequality in (2.8).

The structure of the spectrum for a given intensity of stationary magnetic field, which by virtue of (2.5) defines the value of the parameter $g$, can be obtained with a section of the pertinent planes by the plane of the level $g = \text{const}.$

In Fig. 1, these cross sections are shown for a stainless steel plate with characteristic sizes $A = 10^{-2}$ m, $\eta = 10^{-4}$. In that case, from (2.5) we have $a = 1.7, B^* = |B_1| = 7 \times 10^{-13}$ tesla.

Diagrams $(p, r)$ for given characteristic values of $g$ can be obtained if, in Fig. 1, we consider only the lines enclosed between the points with identical subscripts. The lines $A_1E_1$ represent first solutions, $B_1E_1$ second solutions, $K_1L_1$ third, $C_1P_1$ fourth, $F_1B_1$ fifth, and $B_1D_1$ sixth solutions. The values of the subscript $i$ are defined by the intensity of the magnetic field. For $g < -1.333 \ (B^* < 7 \times 10^{-2} \ \text{tesla})$ we must set $i = 0$, at $-1.333 < g < -1.183 \ (7 \times 10^{-2} \ < B^* \ < 0.2) = 1 = 1$ at $-1.183 < g < -0.583 \ (0.2 < B^* \ < 12.5) = 2 = 2$ at $-0.583 < g < 0.017 \ (12.5 < B^* \ < 790) \ = 3$, at $0.017 < g \ (790 \ < B^*)$. Note that the last two ranges are never achieved in practice; their analysis is of a purely theoretical interest.

3. A solution of the problem of oscillation of a finite plate in a magnetic field can be obtained approximately by using an asymptotic method developed for shell oscillations in a vacuum in $[4]$ and generalized in $[5]$ to the oscillations of shells in a compressible liquid.

According to this method, we perform in $(1.1)-(1.5)$ substitutions of coefficients and variables and a stretching of the scale on the basis of equalities (2.5) (where $B^* = B_1 = B_{32} / \max |B_1|, \eta = \max |B_1| / (2E\mu)^0$) and equalities.
Here $p$, $r$, and $g$ have the same meaning as in §2, while $c$, $d$, and $t$ establish the reciprocal asymptotics of the desired functions; they will be defined later.

These substitutions take Eqs. (1.1)-(1.5) to a form in which with the values of the parameters $(x)=(c, d, k, p, r, g)$ fixed in some way the reciprocal asymptotics of the terms in the equation are defined by the powers $x_n$ of the factors introduced in (1.1)-(1.5): $i_n=\eta^m$, where $x_1=-2p+c+k$, $x_2=2r+t+k$, $x_3=-g-1-p+t+k$, $x_4=-2-4p+d+k$, $x_5=-g-1-p+t+k$, $x_6=a-1+g+t-p-r$, $x_7=g+t$, $x_8=-a-g+p+r$.

For the numbers $\kappa$, formally consistent combinations are chosen that fulfill the requirements of [4]; basically, it is the requirement that a limiting process (at $n \to 0$) be possible such that it would produce a formally consistent limiting equations system, obtained from (1.1)-(1.5) by setting

$$\begin{align*}
j_1 &= j_n=0 \ (x_n=0) \\
j_2 &= j_n=0 \ (x_n=0) \\
j_3 &= j_n=1 \ (x_n=0)
\end{align*}
$$

The number of a variant will be identified by numerals $N$, $n$, and $q$ and denoted as $B(N,n,q)$. The first digit $N$ takes values from 1 to 4 and depends on the choice of the formula $(\alpha, \beta, k_1, k_2)$:

$$
\begin{align*}
N=1 &- c=0, \ d=2, \ k=2p-l, \ r=-p, \ g=g_1, \\
N=2 &- c=l, \ d=0, \ k=-2p, \ r=1-2p, \ g=g_2, \\
N=3 &- c=-\min(2p, 2r), \ d=-\min(2-4p, 2r), \ k=-k, \ r=a-1-p, \ g=\frac{g_1}{2}, \\
N=4 &- c=\max(0, l), \ d=0, \ k=-2p, \ r=1-2p, \ g=g_3
\end{align*}
$$

where $i=i-1-g+t+p$, $g_1=\max(-3+3p, -2a+i+p)$, and at a minimum one (any) of the last two relations corresponding to $N = 2$ contains the equality sign.

Expression (3.2) must be supplemented by formulas defining $t$, $g_1$, $g_2$. Their choice depends on the relationships of the parameters $p$ and $r$, and the third digit $q$ in the number of the variant is chosen accordingly. It has the following values: $q = 1$ at $p+r > a-1$. In this case, $t=1-a-g+p+r$, $g_1=\max(0, p-a)$, $g_2=\min(12p-2a, r-2-1)$. When $p+r < a-1$ here $t=-g$, $g_1=\max(0, p-a)$, $g_2=\min(0, p-a)$.

The relations limiting the values of the parameters $r$ and $g$ in (3.2) (the last two for each $N$) define the portions of the space of parameters $(p, r, g)$ (generally planes limited in some fashion) in which the respective combinations $(c, d, k_1, k_2)$ are noncontradictory.

The transition to limiting systems is done according to rule (3.1), where $n_1, n_2, n_3$ for a given $N$ are

$$
\begin{align*}
N=1 &- n_1=4, 5; n_2=3, 8, 9, 10; n_3=1, 2, 6, 7, 11 \\
N=2 &- n_1=2, 4; n_2=6, 7, 8, 9; n_3=1, 3, 5, 10 \\
N=3 &- n_1=4, 10; n_2=1, 2, 5, 6; n_3=3, 7, 8, 9 \\
N=4 &- n_1=4, 5; n_2=1, 2, 8, 9, 11; n_3=3, 6, 7, 10
\end{align*}
$$

and the factors $j_{n_2}$ must be assigned the values

$$(3.3)$$

This means that in all variants with $q = 1$ the values $(j_n, j_l)=(0, 1)$, and in the variants
with \( q = 2 \) the values \( j_2 = 1 \). In variants corresponding to \( N = 3 \), we will always have \((j_2, j_q) = (1, 1)\), and the digit \( q \) can in that case have the only value \( q = 2 \).

The digit \( n \) which remains undefined in the variant number is determined depending on the relationship of the values of the factors \( j_1, j_2 \) for a given \( N \). These correspondences are specified by the following expressions:

\[
B(1, 1, q) - j_1 = 0, j_2 = 1; \quad B(1, 2, q) - j_1 = 1, j_2 = 0; \quad B(2, 2, q) - j_1 = 0, j_2 = 0; \quad B(2, 3, q) - j_1 = 0, j_2 = 1; \quad B(3, 1, q) - j_1 = 1, j_2 = 0; \quad B(3, 2, q) - j_1 = 1, j_2 = 0; \quad B(3, 3, q) - j_1 = j_2 = 0; \quad B(4, 1, q) - j_1 = j_2 = 0; \quad B(4, 2, q) - j_1 = j_2 = 0; \quad B(4, 3, q) - j_1 = 0, j_2 = 0; \quad B(4, 4, q) - j_1 = 0, j_2 = 1.
\]

4. Limiting equations derived from (1.1)-(1.5) are:

in the variants \( B(N, n, 1) \):

\[
\gamma A F - A \quad (4.1) \\
\Delta \Phi = 0, (\Phi)_{+} - (\Phi)_{-} = F, (\partial(\Phi)/\partial x_2)_{+} - (\partial(\Phi)/\partial x_2)_{-} = 0 \quad (4.2) \\
f = (\partial(\Phi)/\partial x_2)_{-} \quad (4.3)
\]

and in the variants \( B(N, n, 2) \):

\[
\gamma A F + \Delta A = A \quad (4.4)
\]

where \( A \) indicates the respective limiting form of the right-hand side of (1.3). The corresponding limiting forms of (1.1)-(1.2) should be added to the above equations, which can be written as follows (this also applies to \( A \)):

for the variants \( B(1,n,q) \):

\[
L_u \Delta u + \lambda^2 u = -j_1 \beta B, \partial F/\partial x_2 \quad (4.5)
\]

for the variants \( B(2,n,q) \):

\[
L_u \Delta u + \lambda^2 u = -j_2 \beta B, \partial F/\partial x_2 \quad (4.7)
\]

for the variants \( B(3,n,q) \):

\[
L_u \Delta u + \lambda^2 u = -j_1 \beta B, \partial F/\partial x_2 \quad (4.9)
\]

\[
L_u \Delta u + \lambda^2 u = -j_2 \beta B, \partial F/\partial x_2 \quad (4.10)
\]

and for the variants \( B(4,n,q) \):

\[
L_u \Delta u + \lambda^2 u = -j_1 \beta B, \partial F/\partial x_2 \quad (4.11)
\]

\[
L_u \Delta u + \lambda^2 u = -j_2 \beta B, \partial F/\partial x_2 \quad (4.12)
\]

(In (4.5)-(4.12), the summation is done over both subscripts \( j, k = 1, 2 \). For variants \( B(3,n,2), \) in (4.4) we set \( j_3 = 1 \).)

Here and henceforth, in comparing expressions (4.1)-(4.4) with (4.5)-(4.12) it is assumed that the notations \( N, n, \) and \( q \) retained in numbers of the variants can take any of the values as indicated above.

Each of the reduced limiting equations systems generally consists of a number of sub-
systems which must be solved in a certain sequence according to the number of the variant. The sequence is defined by the following expressions:

\[ B(1,1,1) - [(4.5)], -u_n, [(4.1)], -F, (4.6) \rightarrow u_n \]  
\[ B(2,1,1) - [(4.1), (4.3), (4.8)], -u_n, F \]  
\[ B(2,2,1) - [(4.7)], -u_n, [(4.4), (4.7)], -u_n, F; [(4.8)], -u_n \]  
\[ B(4,1,1) - [(4.1)], -F, (4.12) - u_n, [(4.11)], -u_n \]  
\[ B(4,2,1) - [(4.1)], -F; (4.11), (4.12) \rightarrow u_n \]  
\[ B(1,1,2) - [(4.5)], -u_n, [(4.4)], -F, f; (4.8) \rightarrow u_n \]  
\[ B(2,1,2) - [(4.4), (4.5)], -u_n, F, f; (4.3), -u_n \]  
\[ B(2,2,2), B(3,2,2) - [(4.4), (4.7)], -u_n, F, f; [(4.3)], -u_n \]  
\[ B(4,1,2) - [(4.4)], -F, f; (4.12) - u_n, [(4.11)], -u_n \]  
\[ B(3,1,2) - [(4.4)], -F, f; (4.9), (4.11) \rightarrow u_n \]  
\[ B(3,2,2) - [(4.4)], -F, f; (4.10) - u_n, [(4.9)], -u_n \]  
\[ B(3,3,2) - [(4.4)], -F, f; (4.9), -u_n, [(4.10)], -u_n \]  

Relations (4.13) should be combined with expressions [(4.2)] - \( \Phi, (4.3) \rightarrow f, k, m \) run through the values \( k(1,2) \) and \( m(1,2,3) \); the expression \( [(a), (b)] \rightarrow c, d \) means that the variables \( c \) and \( d \) can be defined by simultaneous integration of equations (a) and (b), and the expression \( (a) \rightarrow b \) means that the variable \( b \) is defined by direct actions from formula (a); the subprocesses \( s \) or \( v \) at \([\ldots]\) refers to the region \( S \) or \( V \) in which the equations should be integrated. The sequence of analysis of the equations indicated in (4.13)-(4.15) must be observed.

When integrating on \( S \) equations obtained by the limiting process from (1.1)-(1.3), they must be combined at each point of the edge \( r \) of the plate with two tangential (for (1.1)) and two normal (for (1.2)) boundary conditions of the shell theory and plate theory each \([3]\), and one electromagnetic constraint each; the choice of the latter is discussed below. In integrating Laplace equation (1.4) the condition of boundedness on infinity should also be fulfilled in \( V \).

The electromagnetic condition on \( r \) is chosen so as to eliminate the principal singularity of the problem of integration of the Laplace equation in \( V \), solved either subsequently or within the same limiting subsystem. The choice of this condition in the general case requires additional analysis. In two cases it can be formulated in the initial approximation of accuracy according to the conditions \([2]\): \( \partial F/\partial r = 0 \) (or, which is the same, \( F|\rightarrow 0 \)) for a plate either restrained or having a free end in contact with a non-conducting medium, and \( \partial F/\partial r = 0 \) for a plate restrained in ideally conducting material (here \( \tau \) and \( n \) are tangent and normal to \( r \)). By virtue of (1.6), accurate to \( O(\Delta) \), the equalities \( (\mathbf{n} \cdot m)| = 0 \) and \( (\mathbf{e} \times m)| = 0 \), will hold, representing the absence of a normal current component and of a tangential component of the electric field on the plate edge, respectively.

5. As in \([4,5]\), iterative processes can be constructed for solving problem of the eigenvalues of (1.1)-(1.5); they are based on a consecutive solution of limiting equations corresponding to the different variants, as described in §4. Within each iterative process, a fundamental problem can be identified which is solved first and consists in integration of certain (fundamental) equations; it should admit of nontrivial solutions under homogeneous boundary conditions. Discrepancies in boundary conditions arising in the solution of this problem can be removed by the solution of additional problems.

The variants of limiting equations presented in §4 make it possible to construct a series of iteration processes. We will consider only the fundamental problems, assuming that they can have nontrivial solutions if the pertinent equations contain the frequency parameter \( \lambda \) as one of their coefficients (this condition is fulfilled for all the variants except for \( B(2,3,2) \)). The corresponding fundamental problems are defined by expressions appearing first after the dash in (4.13)-(4.15). Let us spell them out for the following six variants:

\[ 1. L_2 \Phi + \lambda \Phi = 0 \quad B(1, 1, 1) \]  
\[ 2. L_4 \Phi + \lambda \Phi = 0 \quad B(2, 2, 1) \]
A comparison of this result with §2 shows that the domains of variants (5.1) coincide with the domains of the respective numbers in (2.8) of approximate solutions (2.4) and of frequency equations (2.3); the factor \( j' \), has the same meaning as \( f' \) in (2.4), and the formulas (2.4) can be obtained by direct substitution of (2.1) and (2.2) into corresponding equations (5.1). This means, in particular, that for equal relative half-thickness \( \eta \) the asymptotic behaviors of the eigenvalues of the problems of oscillation of a finite plate and an infinite plate given in (5.1) and (2.4) under the same numbers will coincide. One can also say that for a finite plate the eigenvalues will have the same properties as in expressions (2.4); namely, for oscillations of variants 1, 2, and 4, the asymptotic inequality \( \Re e > \Im A \) will take place, indicating a predominant oscillation of the corresponding characteristic solutions in time, while for variants 3, 5, and 6, \( \Im A > \Re e \), meaning that attenuation prevails in time over oscillations. (This assumes that all exact values of the roots of (2.3) are complex, and (2.4) are their initial asymptotic approximations.) The asymptotics of the eigenvalues of problems (5.1) are defined by the last formula of (2.5), where \( r \) is given by the first of the corresponding formulas (2.8).

Returning to the example of §2, we can note that the oscillations for the first two variants can take place also at higher frequencies corresponding to the values \( -1 < \alpha < 0 \), where the shell theory \([4]\) remains valid. The solution of the magnetic elasticity problem, however, should be constructed here with equations different from (1.1)-(1.5); the latter are inapplicable at these frequencies, because their underlying magnetic elasticity hypotheses \([1]\) are violated at \( \alpha = 1 \).

The fundamental equations corresponding to variants \( B(1,2,q) \) and \( B(2,2,q) \), not spelled out here, are rarely used for analysis (in Fig. 1 they are represented by the points \( A_2 \) and \( B_{1,2,2,q} \)), because their domain in the space \( (p, r, q) \) are the intersection lines of the planes defining the domains of the other variants. The approximate solution of the eigenvalue problems for them, generally, is more complex. The fundamental problem of the variant \( B(2,3,2) \) has no eigenvalues, and the variant appears in iterative processes only in the solutions of the additional boundary problems.

We see that in all the variants the factor \( j_y = 0 \). This means that in the initial equations (1.1)-(1.5) the term \( 2hf \) as compared with \( F \) can always be omitted. The resulting equations read \( \eta^{1-p} \). This also applies to formulas (1.6).

Within any sequence (4.13), corresponding to the variants \( B(N,n,l) \), all quantities \( u_m \) and \( F \) are always defined first by solving certain equations on the surface \( S \); one then proceeds to problem (4.2) of defining \( \Phi \) by integrating three-dimensional Laplace equations for a given gradient of the value of the function and for a continuous normal derivation on \( S \). Thus, the solution of any problem of defining \( u_m \) and \( F \) and the quantities expressed through them, which has the property \( \Phi = 0 \), can be found in a two-dimensional statement without considering the field in the surrounding medium. (This property is practically always observed for the first frequencies of transverse oscillations of thin plates of a finite conductivity; it suffices that inequality \( \Phi^3 \Re e > (2 \mu_0 \omega^2) > 0 \) be fulfilled.) Solutions corresponding to the variants \( B(N,n,2) \) for defining all quantities \( u_m \) and \( F \) involve an analysis of three-dimensional problems.

The foregoing results were obtained under the assumption that a stationary magnetic field \( B \) is directed at an angle to the surface \( S \). If some of the components of \( B \) equal zero identically, the classification of the characteristic solution is changed. Thus, from (2.3) it follows that the normal magnetic field \( (B_{01} = B_{02} = 0) \) and the tangential magnetic field \( (B_{03} = 0) \) do not affect the frequencies of transverse and tangential oscillations, respectively. An additional study, which lies beyond the framework of the present paper, has indicated that this influence is manifested in terms of the order \( \delta (\eta^{1-p}) \), that were omitted in the transition from the equations of [1,2] to the equations (1.1)-
its influence is felt at magnetic fields of a much greater intensity than the oblique magnetic fields (generally, by \( n^{-1} \) times).

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